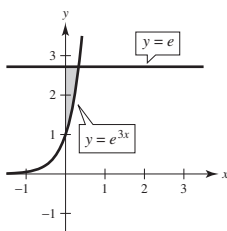


AP<sup>®</sup> Exam Practice Questions for Chapter 7

1.



$$e^{3x} = e$$

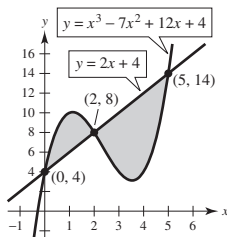
$$3x = 1$$

$$x = \frac{1}{3}$$

$$A = \int_0^{1/3} (e - e^{3x}) dx = \left[ ex - \frac{1}{3} e^{3x} \right]_0^{1/3} = \left( e \cdot \frac{1}{3} - \frac{1}{3} e \right) - \left( 0 - \frac{1}{3} \right) = \frac{1}{3}$$

So, the answer is A.

2.



$$x^3 - 7x^2 + 12x + 4 = 2x + 4$$

$$x^3 - 7x^2 + 10x = 0$$

$$x(x^2 - 7x + 10) = 0$$

$$x(x - 5)(x - 2) = 0$$

$$x = 0, 2, 5$$

$$\begin{aligned} A &= \int_0^2 [(x^3 - 7x^2 + 12x + 4) - (2x + 4)] dx + \int_2^5 [(2x + 4) - (x^3 - 7x^2 + 12x + 4)] dx \\ &= \int_0^2 (x^3 - 7x^2 + 10x) dx + \int_2^5 (-x^3 + 7x^2 - 10x) dx \\ &= \left[ \frac{1}{4}x^4 - \frac{7}{3}x^3 + 5x^2 \right]_0^2 + \left[ -\frac{1}{4}x^4 + \frac{7}{3}x^3 - 5x^2 \right]_2^5 \\ &= \left( 4 - \frac{56}{3} + 20 \right) + \left( -\frac{625}{4} + \frac{875}{3} - 125 \right) - \left( -4 + \frac{56}{3} - 20 \right) \\ &= \frac{253}{12} \end{aligned}$$

So, the answer is C.

$$\begin{aligned} 3. \quad A &= \int_0^{\pi/4} (4 \cos x - 4 \sin x) dx \\ &= [4 \sin x + 4 \cos x]_0^{\pi/4} \\ &= \left[ 4 \left( \frac{\sqrt{2}}{2} \right) + 4 \left( \frac{\sqrt{2}}{2} \right) \right] - [0 + 4(1)] \\ &= 4\sqrt{2} - 4 \\ &= 4(\sqrt{2} - 1) \end{aligned}$$

So, the answer is A.

$$4. \quad 4 \sin x = 2$$

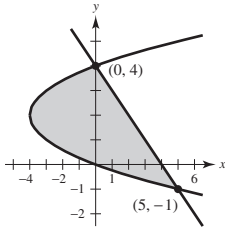
$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}$$

$$\begin{aligned} A &= \int_0^{\pi/6} (2 - 4 \sin x) dx \\ &= [2x + 4 \cos x]_0^{\pi/6} \\ &= \left( \frac{\pi}{3} + 2\sqrt{3} \right) - (0 + 4) \\ &= \frac{\pi}{3} + 2\sqrt{3} - 4 \end{aligned}$$

So, the answer is B.

5.



Write both equations in terms of  $y$  and find the points of intersection.

$$y = -x + 4 \Rightarrow x = 4 - y$$

$$y^2 - 4y = 4 - y$$

$$y^2 - 3y - 4 = 0$$

$$(y - 4)(y + 1) = 0$$

$$y = -1, 4$$

$$\begin{aligned} A &= \int_{-1}^4 [(4 - y) - (y^2 - 4y)] dy \\ &= \int_{-1}^4 (-y^2 + 3y + 4) dy \\ &= \left[ -\frac{1}{3}y^3 + \frac{3}{2}y^2 + 4y \right]_{-1}^4 \\ &= \left( -\frac{64}{3} + 24 + 16 \right) - \left( \frac{1}{3} + \frac{3}{2} - 4 \right) \\ &= \frac{125}{6} \end{aligned}$$

So, the answer is D.

6.  $y = \ln(\sec x)$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\sec x} \cdot \sec x \tan x \\ &= \tan x \end{aligned}$$

$$\begin{aligned} s &= \int_0^{\pi/4} \sqrt{1 + \tan^2 x} dx \\ &= \int_0^{\pi/4} \sqrt{\sec^2 x} dx \\ &= \int_0^{\pi/4} \sec x dx \end{aligned}$$

So, the answer is B.

7.  $y = 4e^{0.5x}$

$$\frac{dy}{dx} = 0.5(4e^{0.5x}) = 2e^{0.5x}$$

$$s = \int_1^4 \sqrt{1 + (2e^{0.5x})^2} dx = \int_1^4 \sqrt{1 + 4e^x} dx$$

So, the answer is A.

8.  $y = \frac{2}{3}x^{3/2}$

$$\frac{dy}{dx} = x^{1/2}$$

$$\begin{aligned} s &= \int_3^8 \sqrt{1 + (x^{1/2})^2} dx \\ &= \int_3^8 \sqrt{1 + x} dx \\ &= \left[ \frac{2}{3}(1 + x)^{3/2} \right]_3^8 \\ &= \frac{2}{3}(27 - 8) \\ &= \frac{38}{3} \end{aligned}$$

So, the answer is B.

9.  $s = \int_1^5 \sqrt{1 + 36x^4} dx$

$$\begin{aligned} &= \int_1^5 \sqrt{1 + (6x^2)^2} dx \\ \Rightarrow \frac{dy}{dx} &= 6x^2 \end{aligned}$$

$$\begin{aligned} \int dy &= \int 6x^2 dx \\ y &= 2x^3 + C \end{aligned}$$

Use (1, 9) to find  $C$ .

$$9 = 2(1)^3 + C$$

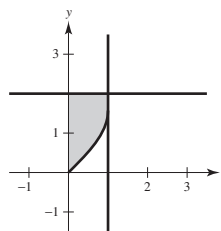
$$9 = 2 + C$$

$$C = 7$$

The equation for the curve is  $y = 2x^3 + 7$ .

So, the answer is A.

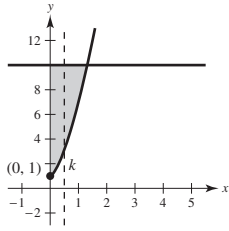
10.



$$\begin{aligned} V &= \int_0^1 [2 - \sin^{-1} x]^2 dx \\ &\approx 2.184 \end{aligned}$$

So, the answer is C.

11. (a)



Find the point of intersection of the graphs.

$$10 = 1 + 6x^{3/2}$$

$$9 = 6x^{3/2}$$

$$\frac{3}{2} = x^{3/2}$$

$$\left(\frac{3}{2}\right)^{2/3} = x$$

$$x \approx 1.3103707$$

$$\int_0^k [10 - (1 + 6x^{3/2})] dx = \int_k^{1.3103707} [10 - (1 + 6x^{3/2})] dx$$

or

$$y = 1 + 6x^{3/2}$$

$$y - 1 = 6x^{3/2}$$

$$\frac{y - 1}{6} = x^{3/2}$$

$$x = \left(\frac{y - 1}{6}\right)^{2/3}$$

$$\int_1^{10} \left(\frac{y - 1}{6}\right)^{2/3} dy = 2 \int_1^{1+6k^{3/2}} \left(\frac{y - 1}{6}\right)^{2/3} dy$$

(b)  $y = 1 + 6x^{3/2}$

$$y' = 6\left(\frac{3}{2}\right)x^{1/2} = 9x^{1/2}$$

$$1 + (y')^2 = 1 + (9x^{1/2})^2 = 1 + 81x$$

$$s = \int_0^1 \sqrt{1 + 81x} dx$$

$$= \frac{1}{81} \int_0^1 (1 + 81x)^{1/2} (81) dx$$

$$= \frac{1}{81} \left[ \frac{2}{3} (1 + 81x)^{3/2} \right]_0^1$$

$$= \frac{2}{243} (82^{3/2} - 1) \approx 6.103$$

(c)  $A = w \cdot h$

$$= [10 - (1 + 6x^{3/2})] \cdot 3[10 - (1 + 6x^{3/2})]$$

$$= 3(9 - 6x^{3/2})^2$$

$$V = 3 \int_0^{1.3103707} (9 - 6x^{3/2})^2 dx \approx 143.289$$

3 pts:  $\begin{cases} 1 \text{ pt: integrand} \\ 2 \text{ pts: limits of integration within correct equation} \end{cases}$

Notes: Use your calculator to find the point of intersection of the two graphs (no work needed for this).

In this intermediate step, round the  $x$ -coordinate of the intersection point to more than three decimal places to use in upcoming integrals.

3 pts:  $\begin{cases} 2 \text{ pts: integral} \\ 1 \text{ pt: answer (no work needed)} \end{cases}$

Notes: Be sure to write down the appropriate definite integral before numerically approximating it on your calculator.

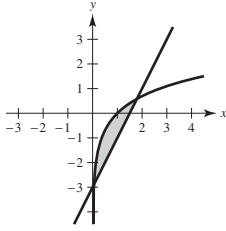
Be sure to round the answer to at least three decimal places to receive credit on the exam.

3 pts:  $\begin{cases} 2 \text{ pts: integral} \\ 1 \text{ pt: answer (no work needed)} \end{cases}$

Notes: Be sure to write down the appropriate definite integral before numerically approximating it on your calculator.

Be sure to round the answer to at least three decimal places to receive credit on the exam.

12. (a)



The graphs intersect at  $x \approx 0.05564832$  and  $x \approx 1.7915369$ .

$$A = \int_{0.05564832}^{1.7915369} [\ln x - (2x - 3)] dx$$

$$\approx 1.471$$

(b)  $V = \pi \int_{0.05564832}^{1.7915369} \left( [\ln(x) + 3]^2 - [(2x - 3) + 3]^2 \right) dx$

$$\approx 18.783$$

(c) Rewrite the equations in terms of  $y$ .

$$y = \ln x \Rightarrow x = e^y$$

$$y = 2x - 3 \Rightarrow x = \frac{1}{2}y + \frac{3}{2}$$

The graphs intersect at  $y \approx -2.888703$  and  $y \approx 0.58307388$ .

$$\pi \int_{-2.888703}^{0.58307388} \left[ \left( \frac{y+3}{2} \right)^2 - (e^y)^2 \right] dy$$

3 pts:  $\begin{cases} 1 \text{ pt: integrand} \\ 1 \text{ pt: limits of integration} \\ 1 \text{ pt: answer (no work needed)} \end{cases}$

Notes: Use your calculator to find the points of intersection of the two graphs (no work needed for this).

In this intermediate step, round the coordinates of the intersection points to more than three decimal places to use in upcoming integrals.

3 pts:  $\begin{cases} 2 \text{ pts: integrand} \\ 1 \text{ pt: limits of integration, constant, answer} \end{cases}$

Notes: Be sure to write down the appropriate definite integral before numerically approximating it on your calculator.

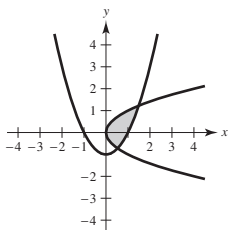
Be sure to round the answer to at least three decimal places to receive credit on the exam.

3 pts:  $\begin{cases} 2 \text{ pts: integrand} \\ 1 \text{ pt: limits of integration and constant} \end{cases}$

Notes: To use the disk method (washer method), set up this integral in terms of  $y$  (using horizontal elements in  $R$ ).

Setting up the volume integral in terms of  $x$  (using vertical elements in  $R$ ) requires the shell method.

13. (a)



Find the points of intersection of the graphs

$$\begin{aligned}
 y = x^2 - 1 &\Rightarrow x = \pm\sqrt{y+1} \\
 \sqrt{y+1} &= y^2 \\
 y+1 &= y^4 \\
 0 &= y^4 - y - 1 \\
 y &\approx -0.724492, 1.2207441
 \end{aligned}$$

$$\begin{aligned}
 A &= \int_{-0.724492}^{1.2207441} (\sqrt{y+1} - y^2) dy \\
 &= \left[ \frac{2}{3}(y+1)^{3/2} - \frac{1}{3}y^3 \right]_{-0.724492}^{1.2207441} \\
 &\approx 1.377
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } V &= \pi \int_{-0.724492}^{1.2207441} \left[ (2 - y^2)^2 - (2 - \sqrt{y+1})^2 \right] dy \\
 &\approx 11.501
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) } 2\pi \int_{-0.724492}^{1.2207441} (y+1)(\sqrt{y+1} - y^2) dy &\text{ or} \\
 \pi \int_0^{0.5248886} \left[ (\sqrt{x} + 1)^2 - (-\sqrt{x} + 1)^2 \right] dx & \\
 + \pi \int_{0.5248886}^{1.4902161} \left[ (\sqrt{x} + 1)^2 - (x^2)^2 \right] dx &
 \end{aligned}$$

$$\begin{aligned}
 &\left\{ \begin{array}{l} 1 \text{ pt: integrand} \\ 3 \text{ pts: } \left\{ \begin{array}{l} 1 \text{ pt: limits of integration} \\ 1 \text{ pt: answer (no work needed)} \end{array} \right. \end{array} \right.
 \end{aligned}$$

Notes: It is strategic to set up this area integral in terms of  $y$ . Setting up the area integral in terms of  $x$  requires the sum of multiple definite integrals.

Use your calculator to find the points of intersection of the two graphs (no work needed for this).

In this intermediate step, round the coordinates of the intersection points to more than three decimal places to use in upcoming integrals.

Be sure to write down the appropriate definite integral before numerically approximating it on your calculator.

Be sure to round the answer to at least three decimal places to receive credit on the exam.

$$\begin{aligned}
 3 \text{ pts: } &\left\{ \begin{array}{l} 2 \text{ pts: integrand} \\ 1 \text{ pt: limits of integration, constant, answer} \end{array} \right.
 \end{aligned}$$

Notes: To use the disk method (washer method), set up this integral in terms of  $y$ .

Setting up this volume integral in terms of  $x$ , via the shell method, requires the sum of multiple definite integrals.

Be sure to write down the appropriate definite integral before numerically approximating it on your calculator.

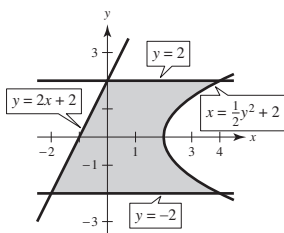
Be sure to round the answer to at least three decimal places to receive credit on the exam.

$$\begin{aligned}
 3 \text{ pts: } &\left\{ \begin{array}{l} 2 \text{ pts: integrand} \\ 1 \text{ pt: limits and constant} \end{array} \right.
 \end{aligned}$$

Notes: It is strategic to set up the shell method here in order to set up a *single* integral.

Setting up this volume integral in terms of  $x$ , via the disk/washer method, requires the sum of multiple definite integrals.

14. (a)



$$\begin{aligned}
 \text{(b) } A &= \int_{-2}^2 \left[ \left( \frac{y^2}{2} + 2 \right) - \left( \frac{y-2}{2} \right) \right] dy \\
 &= \int_{-2}^2 \left( \frac{1}{2}y^2 - \frac{1}{2}y + 3 \right) dy \\
 &= \left[ \frac{1}{6}y^3 - \frac{1}{4}y^2 + 3y \right]_{-2}^2 \\
 &= \left( \frac{19}{3} \right) - \left( -\frac{25}{3} \right) \\
 &= \frac{44}{3}
 \end{aligned}$$

3 pts: sketch of correct region (with boundary curves labeled)

6 pts:  $\begin{cases} 2 \text{ pts: integrand} \\ 2 \text{ pts: limits} \\ 1 \text{ pt: antiderivative} \\ 1 \text{ pt: answer} \end{cases}$

Notes: It is strategic to set up this area integral in terms of  $y$ . Setting up this area integral in terms of  $x$  requires the sum of multiple definite integrals.

The answer does *not* need to be simplified.

$$15. \text{(a) } V = \pi \int_0^2 (\sqrt{x})^2 dx = \pi \int_0^2 x dx = \frac{\pi}{2} [x^2]_0^2 = 2\pi$$

$$\begin{aligned}
 \text{(b) } V &= \pi \int_0^{\sqrt{2}} [2^2 - (y^2)^2] dy \\
 &= \pi \int_0^{\sqrt{2}} (4 - y^4) dy \\
 &= \pi \left[ 4y - \frac{1}{5}y^5 \right]_0^{\sqrt{2}} \\
 &= \frac{16}{5}\pi\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) } \pi \int_0^2 [(\sqrt{x} + 2)^2 - (2)^2] dx \\
 = \pi \int_0^2 [(\sqrt{x} + 2)^2 - 4] dx
 \end{aligned}$$

$$\begin{aligned}
 \text{(d) } A &= \frac{1}{2}bh = \frac{1}{2}(\sqrt{x})\left(\frac{\sqrt{3}}{2}\sqrt{x}\right) = \frac{\sqrt{3}}{4}x \\
 V &= \int_0^{2\sqrt{3}} \frac{x}{4} dx = \left[ \frac{\sqrt{3}}{8}x^2 \right]_0^{2\sqrt{3}} = \frac{\sqrt{3}(2)^2}{8} = \frac{\sqrt{3}}{2}
 \end{aligned}$$

2 pts:  $\begin{cases} 1 \text{ pt: integral} \\ 1 \text{ pt: answer} \end{cases}$

3 pts:  $\begin{cases} 2 \text{ pts: integral} \\ 1 \text{ pt: answer} \end{cases}$

Notes: To use the disk method (washer method), set up this integral in terms of  $y$ .

Setting up the volume integral in terms of  $x$  requires the shell method.

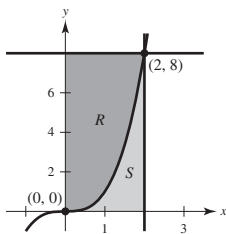
2 pts: integral

Notes: To use the disk method (washer method), set up this integral in terms of  $x$ .

Setting up the volume integral in terms of  $y$  requires the shell method.

2 pts:  $\begin{cases} 1 \text{ pt: integral} \\ 1 \text{ pt: answer} \end{cases}$

16.



$$\begin{aligned}
 \text{(a) } A &= \int_0^2 (8 - x^3) dx \\
 &= \left[ 8x - \frac{1}{4}x^4 \right]_0^2 \\
 &= 16 - 4 \\
 &= 12
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } V &= 2\pi \int_0^2 x(8 - x^3) dx \\
 &= 2\pi \int_0^2 (8x - x^4) dx \\
 &= 2\pi \left[ 4x^2 - \frac{1}{5}x^5 \right]_0^2 \\
 &= \frac{96\pi}{5}
 \end{aligned}$$

or

$$\begin{aligned}
 V &= \pi \int_0^8 (\sqrt[3]{y})^2 dy \\
 &= \pi \int_0^8 y^{2/3} dy \\
 &= \pi \left[ \frac{3}{5}y^{5/3} \right]_0^8 \\
 &= \frac{96\pi}{5}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) } A &= \frac{\pi}{2} \left( \frac{x^3}{2} \right)^2 = \frac{\pi}{8} x^6 \\
 V &= \frac{\pi}{8} \int_0^2 x^6 dx \\
 &= \frac{\pi}{8} \left[ \frac{1}{7}x^7 \right]_0^2 \\
 &= \frac{128\pi}{56} \\
 &= \frac{16\pi}{7}
 \end{aligned}$$

3 pts:  $\begin{cases} 1 \text{ pt: integrand} \\ 1 \text{ pt: limits} \\ 1 \text{ pt: answer} \end{cases}$

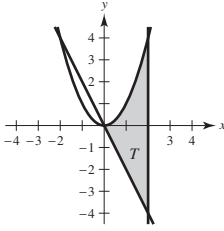
3 pts:  $\begin{cases} 2 \text{ pts: integral} \\ 1 \text{ pt: answer} \end{cases}$

Notes: To use disks, set up this integral terms of  $y$ .

Setting up the volume integral in terms of  $x$  requires the shell method.

3 pts:  $\begin{cases} 2 \text{ pts: integral} \\ 1 \text{ pt: answer} \end{cases}$

17.



$$\begin{aligned}
 \text{(a) } A &= \int_0^2 [x^2 - (-2x)] dx \\
 &= \int_0^2 (x^2 + 2x) dx \\
 &= \left[ \frac{1}{3}x^3 + x^2 \right]_0^2 \\
 &= \frac{8}{3} + 4 \\
 &= \frac{20}{3}
 \end{aligned}$$

3 pts:  $\begin{cases} 1 \text{ pt: integrand} \\ 1 \text{ pt: limits of integration} \\ 1 \text{ pt: answer} \end{cases}$

$$\begin{aligned}
 \text{(b) } V &= \pi \int_0^2 [(-4 - x^2)^2 - (-4 - [-2x])^2] dx \\
 &= \pi \int_0^2 [(16 + 8x^2 + x^4) - (16 - 16x + 4x^2)] dx \\
 &= \pi \int_0^2 (x^4 + 4x^2 + 16x) dx \\
 &= \pi \left[ \frac{1}{5}x^5 + \frac{4}{3}x^3 + 8x^2 \right]_0^2 \\
 &= \pi \left( \frac{32}{5} + 32 + \frac{32}{3} \right) \\
 &= \frac{736\pi}{15}
 \end{aligned}$$

3 pts:  $\begin{cases} 2 \text{ pts: integral} \\ 1 \text{ pt: answer} \end{cases}$

$$\begin{aligned}
 \text{(c) } P &= \sqrt{(2 - 2)^2 + (-4 - 4)^2} + \sqrt{(2 - 0)^2 + (-4 - 0)^2} \\
 &\quad + \int_0^2 \sqrt{1 + (2x)^2} dx \\
 &= 8 + 2\sqrt{5} + \int_0^2 \sqrt{1 + 4x^2} dx
 \end{aligned}$$

3 pts:  $\begin{cases} 2 \text{ pts: arc length integral} \\ 1 \text{ pt: answer (sum of lengths)} \end{cases}$