AP® Exam Practice Questions for Chapter 6

1. To find which graph is a slope field for $\frac{dy}{dx} = y - \frac{x}{5}$, evaluate the derivative at selected points.

At $(0, 1)$, $\frac{dy}{dx} = 1$.

At $(1, 0)$, $\frac{dy}{dx} = -\frac{1}{5}$.

At $(5, 0)$, $\frac{dy}{dx} = -1$,

So, the answer is B.

2.
$$\frac{dP}{dt} = kP$$
$$\int \frac{1}{P}\,dP = \int k\,dt$$
$$\ln|P| = kt + C_1$$
$$P = Ce^{kt}$$

Let $C = 1$ when $t = 0$. Use this and $(15, 3)$ to find k.

$$3 = 1e^{15k}$$
$$\ln 3 = 15k$$
$$\frac{\ln 3}{15} = k$$

So, the answer is B.

3.
$$y' = ky$$
$$\frac{1}{y}y' = k$$
$$\int \frac{1}{y}\,dy = \int k\,dx$$
$$\ln|y| = kx + C_1$$
$$y = Ce^{kx}$$

Use $f(0) = 8$ and $f(6) = 2$ to find C and k.

$$8 = Ce^{k(0)} \Rightarrow C = 8$$
$$2 = Ce^{k(6)}$$
$$2 = 8e^{6k}$$
$$\frac{1}{4} = e^{6k}$$
$$\frac{1}{6}\ln\frac{1}{4} = k$$

Because $f(x) = Ce^{kx} = 8e^{(x/6)\ln(1/4)}$, the answer is A.

4.
$$\frac{dy}{dx} = 2xy^2$$
$$\frac{1}{y^2}\,dy = 2x\,dx$$
$$\int \frac{1}{y^2}\,dy = \int 2x\,dx$$
$$-\frac{1}{y} = x^2 + C$$

Use $y(-1) = 2$ to find C.

$$-\frac{1}{2} = (-1)^2 + C \Rightarrow C = -\frac{3}{2}$$
$$-\frac{1}{y} = x^2 - \frac{3}{2} \Rightarrow y = -\frac{2}{2x^2 + 3}$$
$$y(2) = -\frac{2}{2(2)^2 - 3} = -\frac{2}{5}$$

So, the answer is C.

5.
$$\frac{dy}{dx} = \frac{3y}{x}$$
$$\frac{1}{y}\,dy = \frac{3}{x}\,dx$$
$$\int \frac{1}{y}\,dy = 3\int \frac{1}{x}\,dx$$
$$\ln|y| = 3\ln|x| + C_1$$
$$\ln|y| = \ln|x^3| + C_1$$
$$y = Cx^3$$

Use $y(1) = -1$ to find C.

$$-1 = C(1)^3$$
$$-1 = C$$

The solution of the differential equation is $y = -x^3$.

So, the answer is B.

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6. Evaluate each differential equation for selected values of y.

A: When $y = 2, \frac{dy}{dx} = \frac{20}{3}$.

When $y = 3, \frac{dy}{dx} = 0$.

When $y = 4, \frac{dy}{dx} = -\frac{40}{3}$.

B: When $y = 2, \frac{dy}{dx} = \frac{1}{3}$.

When $y = 3, \frac{dy}{dx} = 0$.

When $y = 4, \frac{dy}{dx} = -\frac{2}{3}$.

C: When $y = 2, \frac{dy}{dx} = \frac{2}{3}$.

When $y = 3, \frac{dy}{dx} = 0$.

When $y = 4, \frac{dy}{dx} = -\frac{4}{3}$.

D: When $y = 2, \frac{dy}{dx} = \frac{20}{3}$.

When $y = 3, \frac{dy}{dx} = \frac{15}{2}$.

When $y = 4, \frac{dy}{dx} = \frac{20}{3}$.

So, the answer is A.

7. Use Euler's Method with $y(0) = -3$, and $h = \frac{1}{3}$ to find $y(1)$.

$y\left(\frac{1}{3}\right) \approx -3 + \frac{1}{3}(-2) = -\frac{11}{3}$

$y\left(\frac{2}{3}\right) \approx -\frac{11}{3} + \frac{1}{3}\left(-\frac{28}{9}\right) = -\frac{127}{27}$

$y(1) \approx -\frac{127}{27} + \frac{1}{3}(-4.753) \approx -6.288$

So, the answer is A.

8. (a) $\frac{dy}{dt} = 0.5y$

$\frac{1}{y}\,dy = 0.5\,dt$

$\int \frac{1}{y}\,dy = \int 0.5\,dt$

$\ln|y| = 0.5t + C_1$

$y = Ce^{0.5t}$

Use $(0, 200)$ to find C.

$200 = Ce^{0.5(0)} \Rightarrow C = 200$

So, $y = 200e^{0.5t}$.

6 pts:
- 1 pt: separation of variables
- 2 pts: antiderivatives
- 1 pt: constant of integration
- 1 pt: uses initial condition
- 1 pt: particular solution (solves for y)

Notes: 3 points max if no constant of integration present

0 points if no separation of variables

(b) $\frac{1}{10}\int_0^{10} 200e^{0.5t}\,dt = \frac{1}{10}\cdot 2\int_0^{10} 200e^{0.5t}(0.5)\,dt$

$= \frac{1}{5}\left[200e^{0.5t}\right]_0^{10}$

$= 40(e^5 - 1)$

≈ 5896.526 bacteria

3 pts:
- 1 pt: definite integral (limits of integration and integrand)
- 1 pt: factor of $\frac{1}{10}$ (for computing the average value)
- 1 pt: answer with units (no work needed)

Notes: Be sure to write down the appropriate definite integral before numerically approximating it on your calculator.

Be sure to round the answer to at least three decimal places to receive credit on the exam.

Importing an incorrect particular solution frompart (a) can still earn the first two points in part (b), but not the answer point.

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9. (a) Use Euler's Method with $y' = \frac{2x}{y}, f(1) = 2,$ and $h = 0.2$ to approximate $f(1.4)$.

$f(1.2) \approx 2 + 0.2\left[\frac{2(1)}{2}\right] = 2.2$

$f(1.4) \approx 2.2 + 0.2\left[\frac{2(1.2)}{2.2}\right]$

So, $f(1.4) \approx 2.2 + 0.2\left[\frac{2.4}{2.2}\right]$.

3 pts: $\begin{cases} \text{2 pts: Euler's Method with two steps} \\ \text{1 pt: answer [approximation of } f(1.4)] \end{cases}$

Note: The answer does *not* need to be simplified. Leaving the answer as $2.2 + 0.2\left[\frac{2(1.2)}{2.2}\right]$ is recommended.

Be sure to write "$f(1.4) \approx$" rather than "$f(1.4) =$." Because this is an approximation, a point may be deducted if an equal sign is used. In general, equating two quantities that are not truly equal will result in a one point deduction on a free-response question.

(b) $\frac{dy}{dx} = \frac{2x}{y}$

$y\,dy = 2x\,dx$

$\int y\,dy = \int 2x\,dx$

$\frac{1}{2}y^2 = x^2 + C_1$

$y^2 = 2x^2 + C$

Use $(1, 2)$ to find C.

$(2)^2 = 2(1)^2 + C$

$4 = 2 + C$

$C = 2$

Because $y^2 = 2x^2 + 2$, the solution is $y = \sqrt{2x^2 + 2}$, where the domain is $(-\infty, \infty)$.

6 pts: $\begin{cases} \text{1 pt: separation of variables} \\ \text{1 pt: antiderivatives} \\ \text{1 pt: constant of integration} \\ \text{1 pt: uses initial condition} \\ \text{1 pt: particular solution (solves for } y) \\ \text{1 pt: states domain of } y \end{cases}$

Notes: 2 points max if no constant of integration present

0 points if no separation of variables

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10. (a) Use Euler's Method, $\frac{dy}{dx} = xy$, $f(1) = 1$, and $h = 0.1$ to find $f(1.2)$.

$f(1.1) \approx 1 + 0.1[(1)(1)] = 1.1$

$f(1.2) \approx 1.1 + 0.1[(1.1)(1.1)]$

So, $f(1.2) \approx 1.1 + 0.1[(1.1)(1.1)]$.

2 pts: { 1 pt: Euler's Method with two steps; 1 pt: answer [approximation of $f(1.2)$] }

Notes: The answer does *not* need to be simplified. Leaving the answer as $1.1 + 0.1[(1.1)(1.1)]$ is recommended.

Be sure to write "$f(1.2) \approx$" rather than "$f(1.2) =$." Because this is an approximation, a point may be deducted if an equal sign is used. In general, equating two quantities that are not truly equal will result in a one point deduction on a free-response question.

(b) $\frac{dy}{dx} = xy$

$$\frac{d^2y}{dx^2} = x\frac{dy}{dx} + y(1)$$
$$= x(xy) + y$$
$$= x^2y + y$$

On the interval $[1, 1.2]$, x is positive. Because $\frac{dy}{dx} = xy$, $\frac{dy}{dx}$ and y have the same sign on $[1, 1.2]$. Because $y = f(1) = 1 > 0$, $\frac{dy}{dx}$ and y are both positive on $[1, 1.2]$ and y is increasing. Therefore, $\frac{d^2y}{dx^2} > 0$, so the solution y is concave upward on this interval and any tangent line approximation will be *below* the curve y. So, the approximation found in part (a) is less than $f(1.2)$.

2 pts: { 1 pt: implicit differentiation to find d^2y/dx^2 (in terms of x and y); 1 pt: answer with justification (appeals to the concavity of y on this interval) }

(c) $\frac{dy}{dx} = xy$

$$\frac{1}{y}\,dy = x\,dx$$
$$\int \frac{1}{y}\,dy = \int x\,dx$$
$$\ln|y| = \frac{1}{2}x^2 + C_1$$
$$y = e^{(1/2)x^2}e^{C_1}$$
$$y = Ce^{(1/2)x^2}$$

Use $(1, 1)$ to find C.

$$1 = Ce^{(1/2)(1)^2}$$
$$1 = Ce^{1/2} \Rightarrow C = e^{-1/2}$$

So, the solution is

$$y = e^{-1/2}e^{(1/2)x^2} = e^{(1/2)x^2 - 1/2} = e^{0.5x^2 - 0.5}.$$

5 pts: { 1 pt: separation of variables; 1 pt: antiderivatives; 1 pt: constant of integration; 1 pt: uses initial condition; 1 pt: particular solution (solves for y) }

Notes: 2 points max if no constant of integration present

0 points if no separation of variables

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11. (a) $\frac{dy}{dx} = \frac{1}{xy}$

$$\frac{d^2y}{dx^2} = \frac{xy(0) - \left[x\frac{dy}{dx} + y\right]}{(xy)^2}$$

$$= -\frac{x\left(\frac{1}{xy}\right) + y}{x^2y^2}$$

$$= -\frac{\frac{1}{y} + y}{x^2y^2}$$

$$= -\frac{1 + y^2}{x^2y^3}$$

At the point $(1, 2)$, $\frac{d^2y}{dx^2} = -\frac{1 + (2)^2}{(1)^2(2)^3} = -\frac{5}{8}$.

2 pts: implicit differentiation to find d^2y/dx^2 (in terms of x and y); evaluates

(b) Find $\frac{dy}{dx}$ at the point $(1, 2)$.

$$\frac{dy}{dx} = \frac{1}{xy} = \frac{1}{(1)(2)} = \frac{1}{2}$$

So, the equation of the tangent line is

$$y - 2 = \frac{1}{2}(x - 1)$$

$$y = \frac{1}{2}x + \frac{3}{2}.$$

$$f(1.1) \approx \frac{1}{2}(1.1) + \frac{3}{2} = 2.05$$

Because $f''(1.1) < 0$, the approximation $f(1.1) \approx 2.05$ is greater than $f(1.1)$.

2 pts:
- 1 pt: finds equation of tangent line using dy/dx at $(1, 2)$ and approximates $f(1.1)$
- 1 pt: answer ("greater than") with reason (appeals to the concavity of f)

(c) $\frac{dy}{dx} = \frac{1}{xy}$

$$y\,dy = \frac{1}{x}\,dx$$

$$\int y\,dy = \int \frac{1}{x}\,dx$$

$$\frac{1}{2}y^2 = \ln|x| + C$$

Use $f(1) = 2$ to find C.

$$\frac{1}{2}(2) = \ln(1) + C$$

$$2 = 0 + C$$

$$2 = C$$

So, the solution is

$$\frac{1}{2}y^2 = \ln|x| + 2$$

$$y^2 = 2\ln|x| + 4$$

$$y = \sqrt{2\ln|x| + 4}.$$

5 pts:
- 1 pt: separation of variables
- 1 pt: antiderivatives
- 1 pt: constant of integration
- 1 pt: uses initial condition
- 1 pt: particular solution (solves for y)

Notes: 2 points max if no constant of integration present

0 points if no separation of variables

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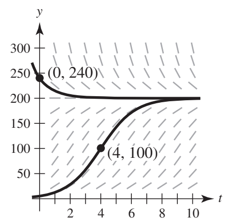
12. (a)



2 pts: solution curves through given points (following slope field)

(b) $\dfrac{dy}{dt} = 0.9y\left(1 - \dfrac{y}{200}\right)$

Use $f(0) = 240$, $h = 0.5$, and Euler's Method to approximate $f(1)$.

$$f(0.5) \approx 240 + 0.5\left[0.9(240)\left(1 - \frac{240}{200}\right)\right] \approx 218.4$$

$$f(1) \approx 218.4 + 0.5\left[0.9(218.4)\left(1 - \frac{218.4}{200}\right)\right] \approx 209.36$$

So, $f(1) \approx 209.36$.

5 pts: $\begin{cases} \text{4 pts: Euler's Method with two steps} \\ \text{1 pt: answer (approximation of } f(1)\text{)} \end{cases}$

Note: Be sure to write "$f(1) \approx$" rather than "$f(1) =$." Because this is an approximation, a point may be deducted if an equal sign is used. In general, equating two quantities that are not truly equal will result in a one point deduction on a free-response question.

(c) When $t \geq 0$, the range of f is $(200, 240)$.

2 pts: answer

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13. (a) At $y = 100$, $\frac{dy}{dt} = \frac{1}{10}y\left(1 - \frac{y}{1000}\right)$

$$= \frac{1}{10}(100)\left(1 - \frac{100}{1000}\right)$$
$$= 10\left(\frac{9}{10}\right)$$
$$= 9.$$

At $y = 200$, $\frac{dy}{dt} = \frac{1}{10}y\left(1 - \frac{y}{1000}\right)$

$$= \frac{1}{10}(200)\left(1 - \frac{200}{1000}\right)$$
$$= 20\left(\frac{8}{10}\right)$$
$$= 16.$$

Because $\frac{dy}{dt} = 9$ when $y = 100$ is less than $\frac{dy}{dt} = 16$ when $y = 200$, the disease is spreading faster when 200 people have the disease.

3 pts: {2 pts: Computes dy/dt at $y = 100$ and $y = 200$; 1 pt: conclusion with reason}

Note: To avoid the risk of an arithmetic mistake, these answers do *not* need to be simplified.

(b) $\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$, where $L = 1000$ and $k = \frac{1}{10}$.

So, $y = \frac{L}{1 + Ce^{-kt}} = \frac{1000}{1 + Ce^{(-1/10)t}}$.

Use $(0, 100)$ to find C.

$$100 = \frac{1000}{1 + Ce^{(-1/10)(0)}}$$
$$1 + C = 10$$
$$C = 9$$

So, a model for the population is $y = \frac{1000}{1 + 9e^{(-1/10)t}}$.

5 pts: {2 pts: general solution of the logistic differential equation; 1 pt: constant of integration; 1 pt: uses initial condition; 1 pt: particular solution}

Note: It is very unlikely that you would be asked to actually solve a logistic differential equation on a free-response question. (This equation is separable, but finding its solution involves a partial fraction decomposition.) However, you may be asked to approximate a solution to a logistic differential equation using Euler's method, a Taylor polynomial, or a slope field.

(c) $\lim_{t\to\infty} y(t) = \lim_{t\to\infty} \frac{1000}{1 + 9e^{(-1/10)t}} = \frac{1000}{1 + 0} = 1000$

1 pt: answer

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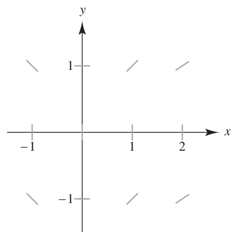
14. (a)



2 pts: slopes of line segments

(b) $\dfrac{dy}{dx} = \dfrac{x}{y^2} = xy^{-2}$

$$\frac{d^2y}{dx^2} = x\left[-2y^{-3}\frac{dy}{dx}\right] + y^{-2}(1)$$

$$= -\frac{2x}{y^3}\left(\frac{x}{y^2}\right) + \frac{1}{y^2}$$

$$= \frac{-2x^2 + y^3}{y^5}$$

2 pts: implicit differentiation to find d^2y/dx^2 (in terms of x and y)

(c) $\dfrac{dy}{dx} = \dfrac{x}{y^2}$

$$y^2\,dy = x\,dx$$

$$\int y^2\,dy = \int x\,dx$$

$$\frac{1}{3}y^3 = \frac{1}{2}x^2 + C$$

Use $y(0) = 2$ to find C.

$$\frac{1}{3}(2)^3 = \frac{1}{2}(0)^2 + C \Rightarrow C = \frac{8}{3}$$

So, the solution is

$$\frac{1}{3}y^3 = \frac{1}{2}x^2 + \frac{8}{3}$$

$$y^3 = \frac{3}{2}x^2 + 8 \Rightarrow y = \sqrt[3]{\frac{3}{2}x^2 + 8}.$$

5 pts:
- 1 pt: separation of variables
- 1 pt: antiderivatives
- 1 pt: constant of integration
- 1 pt: uses initial condition
- 1 pt: particular solution (solves for y)

Notes: 2 points max if no constant of integration present

0 points if no separation of variables

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