AP® Exam Practice Questions for Chapter 2

1. Evaluate each statement.

A: "f is continuous at $x = c$" is a true statement by Theorem 2.1.

B: "$\lim_{x\to c} f(x)$ exists" is a true statement by the definition of a limit.

C: "$f'(c)$ is defined" is a true statement by the definition of a derivative.

D: "$f''(c)$ is defined" could be a false statement because f' may not be differentiable at $x = c$.

So, the answer is D.

2. Evaluate each graph.

A: The graph appears to be $f(x) = -2x$, so $f'(x) = -2$.

B: The graph appears to be $f(x) = x^3$, so $f'(x) = 3x^2$.

C: The graph appears to be $f(x) = x^2$, so $f'(x) = 2x$.

D: The graph appears to be $f(x) = x$, so $f'(x) = 1$.

Because $f'(x) = -2$ is negative for all values of x, the answer is A.

3.
$$y = \frac{6x^4 - 3x^5 + 5x^3}{x^3} = 6x - 3x^2 + 5$$
$$\frac{dy}{dx} = 6 - 6x$$
$$\frac{d^2y}{dx^2} = -6$$

So, the answer is D.

4. $f(x) = \frac{5}{2}\sqrt{x} \Rightarrow f'(x) = \frac{5}{4\sqrt{x}}$

Because $f'(3) = \frac{5}{4\sqrt{3}}$,

$$f'(c) = 2\left(\frac{5}{4\sqrt{3}}\right) = \frac{5}{2\sqrt{3}} = \frac{5\sqrt{3}}{6}.$$
$$\frac{5}{4\sqrt{c}} = \frac{5\sqrt{3}}{6}$$
$$30 = 20\sqrt{3c}$$
$$\frac{3}{2} = \sqrt{3c}$$
$$\frac{9}{4} = 3c$$
$$\frac{3}{4} = c$$

So, the answer is C.

5. The function $h(x) = |2x - 5|$ is continuous at $x = \frac{5}{2}$.

Because $\lim_{x\to(5/2)^-} \frac{|2x - 5|}{x - (5/2)} \neq \lim_{x\to(5/2)^+} \frac{|2x - 5|}{x - (5/2)}$, the function is not differentiable at $x = 5/2$.

So, the answer is A.

6. $y = \sqrt[4]{8x + 3} = (8x + 3)^{1/4}$

$$y' = \frac{1}{4}(8x + 3)^{-3/4}(8) = \frac{2}{(8x + 3)^{3/4}}$$

So, the answer is A.

7. $f(x) = \sin x - \cos x$

$$f'(x) = \cos x + \sin x = \sin x + \cos x$$
$$f''(x) = \cos x - \sin x = -\sin x + \cos x$$
$$f'''(x) = -\cos x - \sin x = -\sin x - \cos x$$
$$f^{(4)}(x) = -\cos x + \sin x = \sin x - \cos x = f(x)$$

Because $n = 4$, the answer is B.

8. $\frac{s(3.2) - s(2.7)}{3.2 - 2.7} = \frac{10.6 - 7.8}{3.2 - 2.7} = \frac{2.8}{0.5} = 5.6$ m/sec

So, the answer is C.

9.
$$2y^3 - 3xy + x^2 = 4$$
$$6y^2\frac{dy}{dx} - 3x\frac{dy}{dx} - 3y + 2x = 0$$
$$(-3x + 6y^2)\frac{dy}{dx} = -2x + 3y$$
$$\frac{dy}{dx} = \frac{-(2x - 3y)}{(-3x + 6y^2)}$$
$$\frac{dy}{dx} = \frac{2x - 3y}{3x - 6y^2}$$

So, the answer is B.

10. $V = \pi r^2 h$

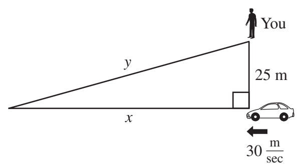
$$\frac{dV}{dt} = \pi\left[r^2\left(\frac{dh}{dt}\right) + 2r\left(\frac{dr}{dt}\right)h\right] = \pi\left(r^2 \cdot \frac{dh}{dt} + 2rh\frac{dr}{dt}\right)$$

When $\frac{dr}{dt} = \frac{1}{3}, \frac{dh}{dt} = \frac{1}{2}, h = 9$, and $r = 4$,

$$\frac{dV}{dt} = \pi\left[(4)^2\left(\frac{1}{2}\right) + 2(4)(9)\left(\frac{1}{3}\right)\right] = 32\pi \text{ cubic centimeters per second.}$$

So, the answer is D.

11.



$$x^2 + 25^2 = y^2$$

$$2x\frac{dx}{dt} + 0 = 2y\frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$$

After 3 seconds, $x = 30(3) = 90$ meters and $y = \sqrt{90^2 + 25^2} = \sqrt{8725} = 5\sqrt{349}$.

Because $\frac{dx}{dt} = 30$ meters per second, $\frac{dy}{dt} = \frac{90}{5\sqrt{349}}(30) \approx 28.906$ meters per second.

So, the answer is B.

12. $s(t) = -t^3 + 2t^2 + \frac{3}{2}$

$s'(t) = -3t^2 + 4t$

$$\text{Average velocity} = \frac{s(4) - s(0)}{4 - 0} = \frac{\left[-(4)^3 + 2(4)^2 + \frac{3}{2}\right] - \left[-(0)^3 + 2(0)^2 + \frac{3}{2}\right]}{4} = \frac{-32}{4} = -8$$

$$s'(t) = -8$$

$$-3t^2 + 4t = -8$$

$$0 = 3t^2 - 4t - 8$$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-8)}}{2(3)}$$

$$t = \frac{4 \pm \sqrt{112}}{6}$$

$t \approx 2.431$ seconds $\quad (t \approx -1.097$ is not in the domain.)

So, the answer is D.

13. (a) $v(t) = 2 + 3.5\cos 0.7t$

$v'(t) = 0 - 3.5(\sin 0.7t)(0.7)$

$= -2.45 \sin 0.7t$

which represents the acceleration of the particle.

6 pts: $\begin{cases} \text{4 pts: computes } v'(t) \\ \text{2 pts: interprets } v'(t) \end{cases}$

(b) $a(5) = v'(5)$

$= -2.45 \sin(0.7 \cdot 5)$

≈ 0.859

At $t = 5$, the acceleration of the particle is about 0.859.

3 pts: answer

14. (a) $$\lim_{h\to 0} \frac{\sqrt{16+h}-4}{h}\left(\frac{\sqrt{16+h}+4}{\sqrt{16+h}+4}\right) = \lim_{h\to 0} \frac{(16+h)-16}{h\left(\sqrt{16+h}+4\right)}$$

$$= \lim_{h\to 0} \frac{h}{h\left(\sqrt{16+h}+4\right)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{16+h}+4}$$

$$= \frac{1}{\sqrt{16}+4} = \frac{1}{8}$$

5 pts:
- 3 pts: scales numerator and denominator by strategic factor (conjugate of numerator)
- 2 pts: answer with analysis (simplifies, cancels out factor of h, evaluates limit)

(b) $$\lim_{h\to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h} = \lim_{h\to 0} \frac{5-5-h}{5(5+h)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-1}{5(5+h)}$$

$$= -\frac{1}{5(5+0)}$$

$$= -\frac{1}{25}$$

4 pts:
- 2 pts: manufacturers a common denominator
- 2 pts: answer with analysis (simplifies, cancels out factor of h, evaluates limit)

Note: A question involving such limits (i.e., evaluating the limit definition of the derivative) is unlikely on the free-response section of the AP Exam.

15. (a)

$$h(x) = \frac{f(x)}{g(x)}$$

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$h'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2}$$

$$= \frac{(5)(1) - (-3)(-2)}{5^2}$$

$$= -\frac{1}{25}$$

3 pts: $\begin{cases} \text{2 pts: uses the Quotient Rule to compute } h'(x) \\ \text{1 pt: answer} \end{cases}$

Note: Leaving the answer as $\frac{(5)(1) - (-3)(-2)}{5^2}$ is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answer on a free-response question. Students are encouraged to not simplify answers on the free-response questions to avoid the risk of making arithmetic mistakes.

(b)

$$j(x) = f(g(x))$$

$$j'(x) = f'(g(x))g'(x)$$

$$j'(2) = f'(g(2))g'(2)$$

$$= f'(5)(-2)$$

$$= (7)(-2)$$

$$= -14$$

3 pts: $\begin{cases} \text{2 pts: uses the Chain Rule to compute } j'(x) \\ \text{1 pt: answer} \end{cases}$

Note: Leaving the answer as $(7)(-2)$ is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answer on a free-response question.

(c)

$$k(x) = \sqrt{f(x)}$$

$$k'(x) = \frac{1}{2\sqrt{f(x)}} \cdot f'(x)$$

$$k'(5) = \frac{1}{2\sqrt{f(5)}} \cdot f'(5)$$

$$= \frac{1}{2\sqrt{4}} \cdot 7$$

$$= \frac{7}{4}$$

3 pts: $\begin{cases} \text{2 pts: uses the Chain Rule to compute } k'(x) \\ \text{1 pt: answer} \end{cases}$

Note: Leaving the answer as $\frac{1}{2\sqrt{4}} \cdot 7$ is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answer on a free-response question.

16. (a) (i) Because $v(t) > 0$ when $0 < t < 1$ and $4.4 < t < 5$, the particle is moving upward on the intervals $(0, 1)$ and $(4.4, 5)$.

(ii) Because $v(t) < 0$ when $2 < t < 4.4$, the particle is moving downward on the interval $(2, 4.4)$.

(iii) Because $v(t) = 0$ when $1 < t < 2$, the particle is at rest on the interval $(1, 2)$.

4 pts:
- 2 pts: (i) answers with explanation, where $v(t) > 0$
- 1 pt: (ii) answer with explanation, where $v(t) < 0$
- 1 pt: (iii) answer with explanation, where $v(t) = 0$

Notes: Though open intervals are desired, open or closed intervals would generally be accepted here.

For (i) and (ii): Because the zero of $v(t)$ on the interval $[4, 5]$ is being determined visually from the given graph, any t-value in the interval $[4.3, 4.5]$ would be acceptable for this zero. (Using exactly $t = 4.4$ in the answers is not required.)

$t = 4.4$ is not included as part of the answer to part (iii) because the directions ask for open intervals where the particle is at rest. (The open interval (4.4, 4.4) would be the empty set.)

(b) (i) When $t = 0.75$, the slope of the line of $v(t) = -2$. So, the acceleration of the particle is -2 feet per second squared.

(ii) When $t = 4.2$, the slope of the line of $v(t) = 5$. So, the acceleration of the particle is 5 feet per second squared.

5 pts:
- 2 pts: (i) answer with analysis (computes slope of this line segment)
- 2 pts: (ii) answer with analysis (computes slope of this line segment)
- 1 pt: units for both (i) and (ii)

Note: Leaving the answers in unsimplified forms, such as $\frac{2 - 0}{0 - 1}$ and $\frac{3 - (-2)}{5 - 4}$, is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answers on a free-response question.

17. (a) $g(x) = f(x)\tan x + kx$

$g'(x) = f'(x)\tan x + \sec^2 x f(x) + k$

Because $\tan \frac{\pi}{2}$ and $\tan \frac{3\pi}{2}$ are undefined and $\sec^2 \frac{\pi}{2}$ and $\sec^2 \frac{3\pi}{2}$ are undefined, the derivative of g will fail to exist when $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

7 pts:
- 3 pts: uses the Product Rule to compute $g'(x)$
- 2 pts: answers
- 2 pts: justification (explaning $\tan x$ and/or $\sec x$ are undefined here)

(b) $g'(x) = f'(x)\tan x + \sec^2 x f(x) + k$

$$g'\left(\frac{\pi}{4}\right) = f'\left(\frac{\pi}{4}\right)\tan \frac{\pi}{4} + \left(\sec \frac{\pi}{4}\right)^2 f\left(\frac{\pi}{4}\right) + k$$

$$6 = (-2)(1) + \left(\sqrt{2}\right)^2(4) + k$$

$$6 = 6 + k$$

$$k = 0$$

2 pts:
- 1 pt: sets up correct equation
- 1 pt: answer (solves for k)

Note: Leaving the answer as $k = 6 - (-2)\tan \frac{\pi}{4} - \left(\sec \frac{\pi}{4}\right)^2(4)$ is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answer on a free-response question.