# AP® Exam Practice Questions for Chapter 1

**1.** $\lim_{x\to\pi} \frac{\sin x}{x} = \frac{\sin \pi}{\pi} = \frac{0}{\pi} = 0$

So, the answer is A.

**2.** $\lim_{x\to -2} \frac{3x^2 + 5x + 7}{x - 4}$

$= \frac{3(-2)^2 + 5(-2) + 7}{(-2) - 4}$

$= \frac{9}{-6}$

$= -\frac{3}{2}$

So, the answer is B.

**3.** Evaluate each statement.

I: As $x$ approaches 3 from the left and right, the function approaches 1. So, $\lim_{x\to 3} \sqrt{x - 2} = 1$ is a true statement.

II: As $x$ approaches 3 from the left and right, the function approaches 0. So, $\lim_{x\to 3} (6 - 2x) = 0$ is a true statement.

III: As $x$ approaches 3 from the left, the function approaches 0 and as $x$ approaches 3 from the right, the function approaches 1. So, the limit $\lim_{x\to 3} f(x)$ does not exist is a true statement.

Because I, II, and III are true statements, the answer is D.

**4.** Evaluate each limit.

I: Using a graphing utility, $\lim_{x\to 1} \frac{x^3 + 1}{x - 1}$ does not exist.

II: $\lim_{x\to 0} \frac{|x|}{x} = \lim_{x\to 0} f(x)$, where $f(x) = \begin{cases} -1, & x < 0 \\ 1, & x \ge 0 \end{cases}$ does not exist because the limits on each side of $x = 0$ do not agree.

III: $\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} 3, & x \le 2 \\ 0, & x > 2 \end{cases}$ does not exist because the limits on each side of $x = 2$ do not agree.

Because the limits of I, II, and III do not exist, the answer is D.

**5.** The domain of

$f(x) = \frac{2}{\sqrt{x - 1}}$ is $\sqrt{x - 1} > 0 \Rightarrow x > 1$.

Because $f$ is not continuous at $x = 1$, the answer is C.

**6.** $\lim_{x\to 5} \left[5f(x) - g(x)\right] = \lim_{x\to 5} 5f(x) - \lim_{x\to 5} g(x)$

$= 5\lim_{x\to 5} f(x) - \lim_{x\to 5} g(x)$

$= 5(10) - (1) = 49$

So, the answer is D.

**7.** Evaluate each statement.

I. Because $\lim_{x\to 2^-} g(x) = 1$ and $\lim_{x\to 2^+} g(x) = 1$, $\lim_{x\to 2} g(x) = 1$.
The statement is true.

II. $\lim_{x\to 2} g(x) = 1 \ne g(2) = 3$
The statement is false.

III. g is continuous at $x = 3$.
The statement is true.

Because I and III are true, the answer is B.

**8.** Evaluate each statement.

A: Because $\lim_{x\to 1^-} f(x) = \infty$ and $\lim_{x\to 1^+} = -\infty$, $\lim_{x\to 1} f(x)$ does not exist. So, $\lim_{x\to 1} f(x) = \infty$ is false.

B: Because $\lim_{x\to 3^-} f(x) = 3$ and $\lim_{x\to 3^+} = 2$, $\lim_{x\to 3^-} f(x) > \lim_{x\to 3^+} f(x)$. So, $\lim_{x\to 3^-} f(x) < \lim_{x\to 3^+} f(x)$ is false.

C: Because $\lim_{x\to 3^-} f(x) = 3$ and $\lim_{x\to 3^+} f(x) = 2$, $\lim_{x\to 3} f(x)$ does not exist. So, $\lim_{x\to 3} f(x) = 1$ is false.

D: Because $\lim_{x\to 0^+} f(x) = 2$ and $\lim_{x\to 3^+} f(x) = 2$, $\lim_{x\to 0^+} f(x) = \lim_{x\to 3^+} f(x)$. So, $\lim_{x\to 0^+} f(x) = \lim_{x\to 3^+} f(x)$ is true.

So, the answer is D.

**9.** The function $f(x) = \dfrac{10}{x^4}$ increases without bound as $x$ approaches 0 from the left and as $x$ approaches 0 from the right. So, the limit is nonexistent.

So, the answer is D.

**10.** Because $\lim_{x\to 1^-} \dfrac{x-1}{\sqrt{x}-1} = 2$ and $\lim_{x\to 1^+} \dfrac{x-1}{\sqrt{x}-1} = 2$, $\lim_{x\to 1} \dfrac{x-1}{\sqrt{x}-1} = 2$.

So, the answer is C.

**11.** (a) $s(1) = 393.1$

$s(2) = 378.4$

Because $s$ is continuous on $[1, 2]$ and $s(2) < 382 < s(1)$, by the Intermediate Value Theorem there exists a time $t$ in the open interval $(1, 2)$ such that $s(t) = 382$ meters.

4 pts: 
- 2 pts: shows $s(2) < 382 < s(1)$ (places 382 in this interval)
- 2 pts: justification (appeals to the continuity of $s$ and/or the Intermediate Value Theorem)

Note: Merely saying "because $s$ is differentiable" or "because $s$ is decreasing" would not earn the justification points.

(b) $s(t) = -4.9t^2 + 398$

$0 = -4.9t^2 + 398$

$t \approx 9.012$

The negative solution does not make sense in the context of the problem. So, the object hits the ground after approximately 9.012 seconds.

1 pt: answer with units

(c)
$$\begin{aligned}
\lim_{t\to 3} \frac{s(t) - s(3)}{t-3} &= \lim_{t\to 3} \frac{\left(-4.9t^2 + 398\right) - \left(-4.9(3)^2 + 398\right)}{t-3} \\
&= \lim_{t\to 3} \frac{-4.9t^2 + 4.9(9)}{t-3} \\
&= \lim_{t\to 3} \frac{-4.9\left(t^2 - 9\right)}{t-3} \\
&= \lim_{t\to 3} \frac{-4.9(t-3)(t+3)}{t-3} \\
&= \lim_{t\to 3} -4.9(t+3) \\
&= -4.9(3+3) \\
&= -29.4 \text{ m/sec}
\end{aligned}$$

4 pts: 
- 2 pts: justification of limit by factoring
- 2 pts: answer with units

**12.** (a) $\lim_{x\to1}[f(x)+4] = \lim_{x\to1} f(x) + \lim_{x\to1} 4$
$= 2 + 4$
$= 6$

1 pt: answer

(b) $\lim_{x\to3^-} \frac{5}{g(x)} = \frac{5}{1} = 5$

1 pt: answer

(c) $\lim_{x\to2}[f(x)g(x)] = (2)(0) = 0$

2 pts:
- 1 pt: justification (indicates the values of both $f(2)$ and $g(2)$; writing $(2)(0)$ is sufficient)
- 1 pt: answer

(d) $\lim_{x\to3} \frac{f(x)}{g(x)-1} = \lim_{x\to3} \frac{(-2x+6)}{(x-2)-1}$
$= \lim_{x\to3} \frac{-2x+6}{x-3}$
$= \lim_{x\to3} \frac{-2(x-3)}{(x-3)}$
$= -2$

5 pts:
- 2 pts: finds linear equations for $f$ and $g$ on $[2, 3]$
- 2 pts: justification of limit by factoring
- 1 pt: answer

Note: The equations for $f$ and $g$ must both be correct to be eligible for the last three points (i.e., the limit must yield the indeterminant form $0/0$).

**13.** (a) Because $\lim_{x\to2^-} f(x) = 1$ and $\lim_{x\to2^+} f(x) = 1$, and the limits are the same, $\lim_{x\to2} f(x) = 1$.

3 pts:
- 2 pts: computes each one-sided limit
- 1 pt: equates the one-sided limits to reach a conclusion about $\lim_{x\to2} f(x)$

(b) Because $f(2) = 3$, $\lim_{x\to2} f(x) = 1$, and $f(2) \neq \lim_{x\to2} f(x)$, $f$ is not continuous at $x = 2$.

4 pts:
- 1 pt: computes $f(2)$
- 2 pts: indicates $f(2) \neq \lim_{x\to2} f(x)$
- 1 pt: reaches correct conclusion

(c) $\lim_{x\to2}[\sin(f(x))] = \sin\left[\lim_{x\to2} f(x)\right] = \sin 1$

2 pts: finds limit

**14.** (a) $f(x) = \dfrac{x^2 + 5x + 6}{2x^2 + 7x + 3} = \dfrac{(x + 2)(x + 3)}{(2x + 1)(x + 3)}$

$= \dfrac{x + 2}{2x + 1},\ x \neq -3$

$f(x)$ has discontinuities at $x = -\dfrac{1}{2}$ and $x = -3$.

4 pts: answers with justification (justify where denominator equals zero)

(b) $\displaystyle\lim_{x\to-3} f(x) = \lim_{x\to-3} \frac{x^2 + 5x + 6}{2x^2 + 7x + 3}$

$\displaystyle= \lim_{x\to-3} \frac{(x + 2)(x + 3)}{(2x + 1)(x + 3)}$

$\displaystyle= \lim_{x\to-3} \frac{x + 2}{2x + 1}$

$= \dfrac{-3 + 2}{2(-3) + 1}$

$= \dfrac{1}{5}$

3 pts: $\begin{cases} \text{2 pts: justification by factoring} \\ \text{1 pt: answer} \end{cases}$

(c) $f(x)$ has a vertical asymptote at $x = -\dfrac{1}{2}$.

2 pts: answer

Note: Including $x = -3$ in the answer (a removable discontinuity) would lose one of the answer points.

**15.** (a) Because $T(x)$ is continuous on $[0, 10]$,

$\lim_{x\to 4} T(x) = T(4) = 172.$

2 pts: 1 pt: answer; 1 pt: justification (appeals to the continuity of $T$)

(b) $$\frac{T(8) - T(3)}{8 - 3} = \frac{164 - 174}{8 - 3} = \frac{-10}{5} = -2$$

The average rate of change is −2°F per minute.

2 pts: 1 pt: justification (evidence of a difference quotient); 1 pt: answer with units

(c) $T(x)$ is continuous and when $x = 6$, $T(x) > 166.5°$ and when $x = 8$, $T(x) < 166.5°$.

So, the shortest interval is $(6, 8)$.

3 pts:
- 1 pt: shows $T(8) < 166.5 < T(6)$ (places 166.5 in this interval)
- 1 pt: answer (an open or closed interval would generally be accepted here)
- 1 pt: justification (appeals to the continuity of $T$ or the Intermediate Value Theorem)

Note: Merely stating "because $T$ is differentiable" or "because $T$ is decreasing" would not earn the justification point.

(d) Because $T(x)$ is continuous, the average rate of change for $6 \le x \le 9$ is

$$\frac{T(9) - T(6)}{9 - 6} = \frac{162 - 168}{9 - 6} = \frac{-6}{3} = -2.$$

So, the tangent line at $x = 8$ has a slope of about $-2$.

2 pts: 1 pt: justification (evidence of a difference quotient on $[6, 9]$); 1 pt: answer

**16.** (a) $f(x) = ax^2 + x - b$ $\qquad f(x) = ax + b$

$f(2) = a(2)^2 + (2) - b \qquad f(2) = a(2) + b$

$= 4a + 2 - b \qquad = 2a + b$

$f$ is continuous at $x = 2$ when $4a + 2 - b = 2a + b$.

$f(x) = ax + b \qquad f(x) = 2ax - 7$

$f(5) = a(5) + b \qquad f(5) = 2a(5) - 7$

$= 5a + b \qquad = 10a - 7$

$f$ is continuous at $x = 5$ when $5a + b = 10a - 7$.

$4a + 2 - b = 2a + b \Rightarrow 2a - 2b = -2$

$5a + b = 10a - 7 \Rightarrow -5a + b = -7$

Multiply both sides of the second equation by 2.

$$\begin{aligned} 2a - 2b &= -2 \\ -10a + 2b &= -14 \\ \hline -8a \quad &= -16 \\ a &= 2 \end{aligned}$$

When $a = 2, b = 5(2) - 7 = 10 - 7 = 3$.

So, $f$ is continuous when $a = 2$ and $b = 3$.

5 pts: 3 pts: equates $4a + 2 - b$ and $2a + b$ (equates the one-sided limits at $x = 2$) and equates $5a + b$ and $10a - 7$ (equates the one-sided limits at $x = 5$)
2 pts: answers (solves the system of equations to determine $a$ and $b$)

(b) $\lim_{x\to 3} f(x) = \lim_{x\to 3}(2x + 3) = 2(3) + 3 = 9$

2 pts: answer from using values of $a$ and $b$ from part (a) and the correct piece of $f$

Note: Incorrect values of $a$ and $b$ may be imported from part (a) as long as $a$ is nonzero (i.e., an answer consistent with values for part (a) can generally earn these two points).

(c)
$$\begin{aligned} \lim_{x\to 1} g(x) &= \lim_{x\to 1} \frac{f(x)}{x - 1} \\ &= \lim_{x\to 1} \frac{2x^2 + x - 3}{x - 1} \\ &= \lim_{x\to 1} \frac{(2x + 3)(x - 1)}{x - 1} \\ &= \lim_{x\to 1}(2x + 3) \\ &= 2(1) + 3 \\ &= 5 \end{aligned}$$

2 pts: 1 pt: justification by factoring
1 pt: answer

Note: To be eligible for these points, the limit must yield the indeterminant form $\frac{0}{0}$ using the values of $a$ and $b$ from part (a) (i.e., incorrect values of $a$ and $b$ generally cannot be imported here).