## Section 2, Part A, Free Response, Technology Permitted

**8.** The rate of growth of the number of bacteria $y$ is given by

$$\frac{dy}{dt} = 0.5y$$

where $t$ is the time in hours and $t \geq 0$. Initially, there are 200 bacteria.

(a) Solve for $y$ at any time $t \geq 0$.

(b) Write and evaluate an expression to find the average number of bacteria in the population for $0 \leq t \leq 10$.

## Section 2, Part B, Free Response, No Technology

**9.** Consider the differential equation $y' = (2x)/y$ with a particular solution in the form of $y = f(x)$ that satisfies the initial condition $f(1) = 2$.

(a) Use Euler's Method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the work that leads to your answer.

(b) Find the particular solution of the given differential equation that passes through (1, 2) and state its domain.

**10.** Consider the differential equation $dy/dx = xy$.

(a) Let $y = f(x)$ be the function that satisfies the differential equation with initial condition $f(1) = 1$. Use Euler's Method, starting at $x = 1$ with a step size of 0.1, to approximate $f(1.2)$. Show the work that leads to your answer.

(b) Find $d^2y/dx^2$. Determine whether the approximation found in part (a) is less than or greater than $f(1.2)$. Justify your answer.

(c) Find the particular solution of the given differential equation that passes through (1, 1).

**11.** Let $y = f(x)$ be a particular solution of the differential equation

$$\frac{dy}{dx} = \frac{1}{xy}$$

with $f(1) = 2$.

(a) Find $d^2y/dx^2$ at the point (1, 2).

(b) Write an equation for the line tangent to the graph of $f$ at (1, 2) and use it to approximate $f(1.1)$. Is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

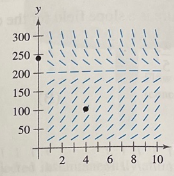
(c) Find the solution of the given differential equation that satisfies the initial condition $f(1) = 2$.

**12.** Consider the differential equation

$$\frac{dy}{dt} = 0.9y\left(1 - \frac{y}{200}\right).$$

Let $y = f(t)$ be the particular solution of the differential equation with $f(0) = 240$.

(a) A slope field for this differential equation is given below. Sketch possible solution curves through the points (0, 240) and (4, 100).




(b) Use Euler's Method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$.

(c) What is the range of $f$ for $t \geq 0$?

**13.** At any time $t \geq 0$, the rate of the spread of a disease is modeled by the differential equation

$$\frac{dy}{dt} = \frac{1}{10}y\left(1 - \frac{y}{1000}\right)$$

where $y$ is the number of people who have the disease. In an isolated town of 1000 inhabitants, 100 people have the disease at the beginning of the week.

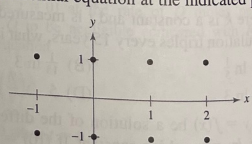
(a) Is the disease spreading faster when 100 people have the disease or when 200 people have the disease? Explain your reasoning.

(b) Write a model for the population $y = f(t)$ at any time $t \geq 0$.

(c) What is $\lim_{t\to\infty} y(t)$?

**14.** Consider the differential equation $dy/dx = x/y^2$, where $y \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the indicated points.




(b) Find $d^2y/dx^2$ in terms of $x$ and $y$.

(c) Find the particular solution of the given differential equation that satisfies the initial condition $y(0) = 2$.