Section 2, Part A, Free Response, Technology Permitted

14. A particle travels along the x-axis for times $0 \le t \le 4$. The velocity of the particle is given by

$$v(t) = \sin\left(\frac{5\pi}{2}e^{-t/\pi}\right).$$

When $t = 0$, the particle is 2 units to the right of the origin.

(a) During what time intervals on $0 \le t \le 4$ is the particle traveling to the left?

(b) Find the average velocity of the particle on $0 \le t \le 4$.

(c) Is the speed increasing or decreasing at time $t = 2$?

(d) For the time interval $0 \le t \le 4$, what is the farthest distance to the right that the particle travels from the origin?

15. Let f be a function defined by $f(x) = \frac{1}{2}(e^x + e^{-x})$.

(a) Find, if any, the x-coordinates of all relative minimum values of f.

(b) Find the average value of f for $-1 \le x \le 1$.

(c) Find, if any, the x-coordinates of all inflection points of the graph of f.

16. The function f is defined by $f(x) = 1/\sqrt{4 - x^2}$ for $-2 < x < 2$. Let g be the function defined by

$$g(x) = \begin{cases} f(x), & -2 < x \le 0 \\ x + \frac{1}{2}, & 0 < x \le 2 \end{cases}.$$

(a) Is g continuous at $x = 0$? Use the definition of continuity to explain your answer.

(b) What is the area of the region bounded by f, the x-axis, the y-axis, and the line $x = 1$?

(c) Find the value of $\int_{-1}^{1} g(x)\,dx$.

17. Water floods into a basement at a rate given by $f(t) = -2\sqrt{t} + 5$. At the same time, water is pumped out of the basement at a rate given by $g(t) = 5(1 - e^{-0.5t})$. Both $f(t)$ and $g(t)$ are measured in cubic feet per hour, and t is the time in hours, where $0 \le t \le 5$. When $t = 0$, there are 15 cubic feet of water in the basement.

(a) How many cubic feet of water enter the basement from $t = 0$ to $t = 2$ hours?

(b) How fast is the water level changing at time $t = 2$ hours?

(c) At what time t is the volume of water at a maximum? How much water is in the basement at that time?

18. Two towers of equal height are spaced 366 feet apart. A cable suspended between the two forms a catenary whose height above the ground is given by $f(x) = \frac{125}{2}(e^{x/250} + e^{-x/250})$, where $x = 0$ is at the point on the ground halfway between the two towers.

(a) What is the height of each tower (rounded to the nearest foot)? (Assume the cable is anchored to the *tops* of the towers.)

(b) Evaluate $f'(100)$. Explain what $f'(100)$ is in the context of the problem.

(c) What is the average height of the cable?

(d) What is the slope of the cable at the rightmost point where the height of the cable is equal to the average height of the cable?

Section 2, Part B, Free Response, No Technology

19. Let $f(x) = \arccos x$ and let $g(x) = x^2$. Define $h(x) = f(g(x))$.

(a) At what value(s) of x does h have a relative maximum? Justify your answer.

(b) Write, but do not evaluate, an integral expression that gives the area of the region bounded by the graph of h and the horizontal line $y = \pi/3$.

(c) Evaluate $(f^{-1})'(\pi/3)$.

20. Let $f(x) = e^x - x$.

(a) Find the critical values of f. Classify each of these values as a relative minimum, a relative maximum, or neither. Justify your answer.

(b) Write an equation of the line tangent to the graph of f at $x = 1$.

(c) Given $\int_0^a f(x)\,dx = f'(a)$, find a.

21. The table shows values of f and its derivative f' at selected values of x. The function f is twice differentiable for all real numbers.

x	1	2	3	4	5
$f(x)$	3	-1	5	-2	4
$f'(x)$	1	3	-2	4	-2

(a) Use the table to estimate the value of $f'(1.5)$.

(b) Given $1 < c < 3$, explain why there must be a value c for which $f'(c) = 1$.

(c) Given f has an inverse function, find an equation of the line tangent to the graph of $y = f^{-1}(x)$ at $x = 5$.